বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - V

Subject : PHYSICS

Paper : DSE 1-T & P

Full Marks : 60 (Theory - 40 + Practical - 20)

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[ADVANCED MATHEMATICAL PHYSICS-I]

(Theory)

Group-A

A. Answer any *three* of the following questions : 12×3=36

1. (a) Obtain Laplace transformation of $t^2 e^t \sin 4t$.

   (b) Find Laplace transformation of $\int_0^t e^{-2t} t \sin^3 t.dt$. 6+6

2. (a) Express the following function in terms of unit step functions and find its laplace transformation $f(t) = \begin{cases} 8 \; ; \; t > 2 \\ 6 \; ; \; t < 2 \end{cases}$

(b) Prove that $\hat{L}\,f(t)\cup(t-o)=e^{-as}\hat{L}\left[f(t+a)\right]$.

3. (a) Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and other is skew-symmetric in a pair of covariant or contravariant indicies.

(b) Show that symmetric and antisymmetric property of a tensor is conserved under a transformation of coordinates. 6+6

4. (a) If A^{i} are the components of an absolute contravariant tensor of rank one, show that $\frac{\partial A_i}{\partial x_j}$ are the components of mixed tensor.

(b) Show that the expression *A (i, j, k)* is a tensor if its inner product with an arbitrary tensor B_k^{jl} is a tensor. 6+6

5. A resistance *R* in series with inductance *L* is connected with e.m.f *E(t)*. The current is given by $L\frac{di}{dt}+Ri=E(t)$.
If the switch is connected at $t = 0$ and disconnected at $t = a$; Find current *i* in terms of '*t*'. 12

6. Find the inverse Laplace Transformation of $\frac{1}{(s+1)(s^2+1)}$. 12

Group-B

B. Answer any *two* of the following questions : 2×2=4

7. Find the Laplace transform of $\sin t\cos t$.

8. Evaluate : (i) $\delta_j^i\delta_k^i$ (ii) $\delta_j^i\,\delta_k^j\,\delta_l^k$.

9. Find Laplace transform of $\frac{\sin 2t}{t}$.

10. Show that velocity of a Fluid at any point is a contravariant tensor of rank one.

(Practical)

Group-A

A. Answer any *one* of the following questions : 15×1=15

1. Find Eigenvalues and Eigenvctors of the matrix

$$\begin{pmatrix} 1 & -i & 3+4i \\ i & 2 & 4 \\ 3-4i & 4 & 3 \end{pmatrix}.$$ 15

2. Determine Principal axes of moment of inertia through diagonalization. 15

3. If $A=\begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, find the products, AB and BA, show that $AB \neq BA$. 15

Group-B

B. Compulsory Questions : (Answer any *one* question) 5×1=5

4. Write down position and Momentum operator and Calculate their commutator. 5

6. Find $P_5(x)$ using the recursion relation for Legendre polynomial
$n P_n(x)=(2n-1)x P_{n-1}(x)-(n-1)P_{n-2}(x)$ with $P_0(x)=1$.

7. Lab Note book / Viva.

OR

[APPLIED DYNAMICS]

(Theory)

Group-A

A. Answer any *three* of the following questions : 12×3=36

1. (a) What are Phase point, Phase space, Phase portrait and trajectories? Explain with examples. 6

   (b) Sketch the phase portrait corresponding to $\dot{x} = x\cos x$ and determine the stability of all the fixed points. 3

   (c) Give two examples of non-uniform oscillator from each in the field of condensed matter physics, mechanics and biology. 3

2. (a) Explain population growth. Hence establish logistic equation and discuss the relations between population and carrying capacity. 2+2+3

   (b) Discuss biological validity of the logistic equation. 2

   (c) Classify the fixed points of the logistic equation, using linear stability analysis, and find the characteristics time scale in each case. 3

3. (a) Explain fractal geometry. 2

   (b) Make a comparison between self-similar fractal structure and deterministic fractal structure. 3

   (c) Suppose that f has a stable p-cycle containing the point x_0. Show that the Lyapunov exponent $\lambda < 0$. If the cycle is superstable, show that $\lambda = -\infty$. 3

   (d) What is the principle of Sierpinski gasket? How do you use it? 2+2

4. (a) The growth of a cancerous tumors can be modelled by the law $\dot{N} = -aN\ln(bN)$, where $N(t)$ is proportional to the number of cells in the tumor, and a, b are parameters (a, $b > 0$).

   (i) Interpret a and b biologically.

   (ii) Sketch the vector field and the graph $N(t)$ for various initial values. 2+2

(b) Show that the non-linear Duffing equation $\ddot{x} + x + \varepsilon x^3 = 0$ has a non-linear center at the origin for all $\varepsilon > 0$. If $\varepsilon < 0$ show that all trajectories near the origin are closed. 2+2

(c) What are chaotic maps? How do you identify a chaotic system? 2+2

5. (a) Explain the continuum hypothesis-concept of fluid parcel or fluid elements. 4

(b) The velocity $v(t)$ of a skydiver falling to the ground is governed by $m\dot{v} = mg - kv^2$, where m is the mass of the skydiver and $k > 0$ is a constant related to the amount of air resistance.

(i) Obtain the analytical solution for $v(t)$, assuming that $v(0) = 0$.

(ii) Find the limit of $v(t)$ as $t \to \infty$. 3+1

(c) Write short note on transport properties of a fluid—Thermal conductivity and mass diffusivity. 2+2

6. (a) Define pressure drag and viscous drag. 2+2

(b) Find the pressure field in the inviscid incompressible flow with velocity field $u = (nx, -ny, 0)$. 3

(c) Write down Navier-Stokes equations of incompressible fluid flow. 2

(d) Explain 'pseudo plastic fluid', 'dilatant fluid' and 'Bingham plastic fluid'. 3

Group-B

B. Answer any ***two*** **of the following questions :** **2×2=4**

7. What do you mean by fixed points?

8. Define Population models.

9. What is 'Characteristics time scale'?

10. What do you mean by Cobweb iteration?

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(Practical)

Group-A

A. Answer any *one* of the following questions : 15×1=15

1. Determination of coupling coefficient of coupled pendulums :
   - (a) Theory 3
   - (b) Working formula 2
   - (c) Data collection procedure 8
   - (d) Remarks 2
2. Determination of coupling and damping coefficient of damped coupled oscillator.
   - (a) Theory and required formula 4
   - (b) Data collection procedure 7
   - (c) Discussion 2
   - (d) Calculation of Error 2
3. Calculation of population modes e.g. exponential growth and decay, logistic growth, species competition, predator-prey dynamics, simple genetic circuits.
   - (a) Theory 3
   - (b) Working formula 3
   - (c) Data collection procedure 7
   - (d) Discussions 2

Group-B

B. Compulsory Questions : (Answer any *one* question) 5×1=5

4. Compute the computational visualization of fractal formation of Fractals in nature-earthquakes. 5
5. Compute the computational flow visualization of streamlines. 5
6. Compute the computational visualization of trajectories in the Sinai Billiard. 5
7. Lab Notebook / Viva-Voce. 5

OR

[ATMOSPHERIC PHYSICS]

(Theory)

Group-A

A. Answer any *three* of the following questions : 12×3=36

1. (a) What is the difference between barotropic fluid and baroclinic fluid? 2

   (b) Write down the advantages of using isobaric co-ordinate system over height co-ordinate system. 3

   (c) The tendency of cyclones to move pole ward is known as 'beta drift'. Suggest the basis for the above tendency. 2

   (d) Let the air whirl at 90 m/s around a tornado having a radius of 120m. What would be the slope of an isobaric surface associated with the tornado? 2

   (e) What do you mean by a depression strom? 2

   (f) What is Psws No 2? 1

2. (a) Define Aerosol. What are primary aerosols and secondary aerosols? 2+2

   (b) What do you mean by spontaneous nucleation of condensation? Why it is occurred? 2+2

   (c) Write down the definition of the following : Freezing nuclei, Contact nuclei, Deposition nuclei and ice nuclei. 4

3. (a) Derive Rayleigh scattering formula. 5

   (b) Write down difference between Rayleigh scattering and Mie scattering. 2

   (c) Give the examples of Mie scattering. Is Mie scattering isotropic? 1+2

   (d) Assuming Rayleigh scattering by air molecules, find the relative efficiencies with which red light $(\lambda = 0.64\,\mu m)$ and blue light $(\lambda = 0.47\,\mu m)$ are scattered. 2

4. (a) Define the term 'fastest wave'. 2

(b) Derive the equation for propagation of atmospheric gravity waves in a non-homogeneous medium. 5

(c) The barometric Rossby wave is also known as constant absolute vorticity trajectories. What is the direction of movement of Rossby wave? 2

(d) A background jet stream of speed 60 m/s meanders with 6000 km wavelength ad 1500 km amplitude, angle at 45ºN. Find the phase speed relative to the ground of the barometric Rossby wave. 3

5. (a) What is meterology radar? Write down the different types of radar. 1+2

(b) Write down radar equation. How signal processing and detection are done in radar system? Explain with block diagram. 2+4

(c) What is a Lidar? Write down applications of Lidar to study atmospheric phenomena. 1+2

6. (a) Write down the most general relationship between vorticity and circulation. 2

(b) Explain absolute vorticity and potential vorticity. 2+2

(c) If wind rotates as a solid body around the center of a low pressure system and the tangential velocity is 10 m/s at a radius of 200 km, find the relative vorticity. 3

(d) For a small element of area in 2D show that the vertical component of the vorticity of the fluid in the area is equal to the circulation around the perimeter per unit area. 3

Group-B

B. Answer any *two* of the following questions : 2×2=4

7. What are cyclones and anticyclones?

8. Write down Bouguert-Lambert law.

9. What do you mean by enhanced greenhouse effect?

10. What is Doppler radar used for?

(Practical)

Group-A

A. Answer any *one* of the following questions : 15×1=15

1. Compute numerical simulation for Rossby waves using dispersion relations. 15

2. Time series analysis of temperature using long term data over metropolitan cities in India. 15

3. Processing of Radiosonde data and its interpretation in terms of atmospheric parameters using vertical profiles in different regions of the globe. 15

Group-B

B. Compulsory Questions : (Answer any *one* question) 5×1=5

4. Process online data of VHF radar. 5

5. Process online data of X-band radar. 5

6. Online processing of LIDAR data. 5

7. Practical Note book / Viva-voce. 5

OR

[CLASSICAL DYNAMICS]

(Theory : Marks - 60)

Group-A

A. Answer any *four* of the following questions : 12×4=48

1. (a) Derive Lagrange's equation in presence of non-conservative forces. 4

   (b) Find the Lagrange's equation of motion for a LC circuit and also deduce the time period. 4

   (c) A particle moves in a plane under the influence of a force, acting towards a center of force, whose magnitude is $F = \frac{1}{r^2}\left(1 + \frac{2r\ddot{r} - \dot{r}^2}{c^2}\right)$. Where r is the distance of the particle from the center of force. Find the generalised potential that will result in such a force, and write down the Lagrangian for the particle. 4

2. (a) A particle moving under central force field, set up the equation of motion in terms of plane polar co-ordinates. 3

   (b) Show that the effective potential of a particle in a central force field is given by $V_{eff}(r) = V(r) + \frac{L^2}{2mr^2}$. Where L is the angular momentum. 4

   (c) In an attractive inverse square central field, the equation of motion of a particle (mass m) is given by $\dot{\bar{p}} = -\frac{k}{r^2}\hat{r}$ (k is positive constant). Show that $\dot{\bar{p}} \times \bar{L} = \frac{d}{dt} mk\hat{r}, \bar{L}$ being constant of motion.

   Hence show that Laplace-Runge-Lenz vector $\bar{R} = \bar{p} \times \bar{L} - mk\hat{r}$ is constant of motion. 5

3. (a) What do you mean by stable and unstable equilibriums? The potential energy of a particle is given by $V = 3x^4 - 8x^3 - 6x^2 + 24x$. Find the points of stable and unstable equilibrium. 2+2

   (b) Examine whether the cylindrical polar co-ordinates (ρ, θ, z) represent a set of generalised co-ordinates. 4

(c) Show that the Lagrangian and Newtonian equations are identical if the generalised co-ordinates are the rectangular co-ordinates. 4

4. (a) What do you mean by normal modes of vibration? Explain the meaning of normal co-ordinates and normal frequencies. 2+(2+2)

(b) Show that when the kinetic and potential energies are expressed in terms of normal co-ordinates, both kinetic and potential energies are homogeneous quadratic functions. 3

(c) Determine the normal mode frequency of the Lagrangian given by $L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\left(\omega_1^2 x^2 + \omega_2^2 y^2\right) + \alpha xy$. 3

5. (a) Discuss relativistic Dopller effect and derive relation for longitudinal Doppler effect. Hence define red shift and blue shift. (2+4)+2

(b) A meson of rest mass m_0 comes to rest and disintegrates into a muon of rest mass μ and a neutrino o zero mass. Show that the kinetic energy of the motion of muon is $T = \frac{1}{2m_0}\left(m_0 - \mu\right)^2 c^2$. 4

6. (a) What do you mean by Minkowski space? Explain. 2

(b) What are space like and time like intervals? What is their significance? 2+2

(c) Show that the quantity $\left(\vec{p}^2 - \frac{E^2}{c^2}\right)$ is a Lorentz invariant. 3

(d) Is the second law of motion $\bar{F} = m\bar{a}$ is always valid in special theory of relativity? 2

(e) What are Tachyons? 1

7. (a) Derive the equation of continuity for 2D flow in polar co-ordinates. 5

(b) In a 2D fluid motion, the velocity components in cartesian coordinates are given by $u = -\frac{ay}{x^2 + y^2}, v = \frac{ax}{x^2 + y^2}, w = 0$. ($a$ is constant). Show that the flow is possible. Also find the equation of stream lines. 2+2

(c) A cylinder of cross-sectional area A is full of liquid (density ρ) between radius r_1 and r_2. It revolves about a vertical axis with constant angular velocity ω. Show that the total centrifugal force impressed is $\frac{\rho A}{2g}\left(v_2^2 - v_1^2\right)$, where v_2 and v_1 are the tangential velocities at $r = r_1$ and $r = r_2$, respectively. 3

8. (a) Set up Euler's equation of hydrodynamic for an incompressible fluid. 4

(b) Use Euler's equation and derive Bernoulli's theorem for steady streamline motion of an incompressible fluid. 5

(c) The velocity field of the motion of an incompressible fluid is $\vec{v} = \left(x^2y + y^2\right)\hat{i} - xy^2\hat{j}$. Determine the vorticity vector and pressure gradient in *x*- and *y*-direction on the horizontal plane. 3

Group-B

B. Answer any *six* of the following questions : 2×6=12

9. Discuss the superiority of Lagrangian approach over Newtonian approach.

10. Define cyclic co-ordinate with example.

11. Write down the Lagrangian for a particle of mass *m* falling freely under gravity near the surface of the earth.

12. What do you meant by 'small amplitude oscillation'?

13. Explain the terms : Proper time and relativistic time.

14. Two photons approach each other, what is their relative velocity?

15. Define Four vectors with examples.

16. What is stream-line motion? Is energy conserved along a streamline?

17. Write down the physical significance of the Reynolds Number.

18. Verify if $\vec{V} = -\frac{2xyz}{x^2+y^2}\hat{i} + \frac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2}\hat{j} + \frac{y}{x^2+y^2}\hat{k}$ is a possible motion for an incompressible ideal fluid or not.

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